

# Isospin symmetry breaking within the HLS Model: A full ( $\rho$ , $\omega$ , $\phi$ ) mixing scheme

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**Abstract.** We study the way isospin symmetry violation can be generated within the Hidden Local Symmetry (HLS) Model. We show that isospin symmetry breaking effects on pseudoscalar mesons naturally induces correspondingly effects within the physics of vector mesons, through kaon loops. In this way, one recovers all features traditionally expected from  $\rho - \omega$  mixing and one finds support for the Orsay phase modelling of the  $e^+e^- \rightarrow \pi^+\pi^-$  amplitude. We then examine an effective procedure which generates mixing in the whole  $\rho$ ,  $\omega$ ,  $\phi$  sector of the HLS Model. The corresponding model allows us to account for all two body decays of light mesons accessible to the HLS model in modulus and phase, leaving aside the  $\rho \rightarrow \pi\pi$  and  $K^* \rightarrow K\pi$  modes only, which raise a specific problem. Comparison with experimental data is performed and covers modulus and phase information; this represents 26 physics quantities successfully described with very good fit quality within a constrained model which accounts for SU(3) breaking, nonet symmetry breaking in the pseudoscalar sector and, now, isospin symmetry breaking.

## 1 Introduction

Despite the large difference in the  $u$  and  $d$  current quark masses, isospin violation in the strong interaction is typically at the order of a few percent, such as the  $\pi^\pm - \pi^0$  mass difference. This is because the scale is set not by  $(m_u - m_d)/(m_u + m_d)$  but  $(m_u - m_d)/m_s$  [1]. Interest in the contribution of isospin violation is therefore usually confined to systems where both theoretical (or at least phenomenological) and experimental precision are high; for example  $a_\mu$  [2], CP violation in  $B \rightarrow 2P$  (where  $P \equiv$  pseudoscalars) and other CKM-matrix systems [3–6], the pion form-factor [7,8] and various aspects of charge symmetry violation in the  $NN$  system [9].

However, in  $e^+e^- \rightarrow \pi^+\pi^-$  the isospin violating process of  $\rho - \omega$  mixing produces a large effect on the interaction. This is due to both the isospin independence of the initial vertex (the coupling of the  $\omega$  to the photon is only a third of the coupling of the  $\rho^0$  to the photon) and narrow width of the  $\omega$  (in the region of the  $\omega$  resonance the cross-section is approximately 40% larger than it would be without  $\rho - \omega$  mixing). Therefore any strongly interacting system where the  $\rho^0$  and  $\omega$  have significant (if not necessarily large) production amplitudes can expect a similar enhancement in  $\pi^+\pi^-$  pair production in the  $\rho - \omega$  interference region. Lipkin realised this would ap-

ply to various decays in the  $B$  system [10]. Building on this, Enomoto and Tanabashi discovered a decay channel that would show a sizeable direct CP asymmetry,  $B^- \rightarrow \rho^-(\rho^0/\omega) \rightarrow \rho^-\pi^+\pi^-$ . Here the penguin term exists only for  $B^- \rightarrow \rho^-\omega$  not the  $B^- \rightarrow \rho^-\rho^0$  and the necessary penguin/tree interference arises through  $\rho - \omega$  mixing with the strong phase courtesy of the  $\omega$  propagator [11] (for further details see [12]). This gives a renewed interest to the description of isospin symmetry breaking.

Having just said that  $\rho - \omega$  mixing can lead to large effects, it is important to explain the quoted figure ( $\sim 2\%$ ) for the  $\omega \rightarrow 2\pi$  branching fraction. The pion form factor can be defined (and a definition is a useful thing) through [13]

$$F_\pi(s) = F_\rho(s) \left( 1 + \frac{f_{\omega\gamma}}{f_{\rho\gamma}} \frac{\tilde{\Pi}_{\rho\omega}}{s - m_\omega^2 + im_\omega\Gamma_\omega} \right). \quad (1)$$

Though the mixing amplitude  $\tilde{\Pi}_{\rho\omega} \simeq -4300$  MeV<sup>2</sup> is small compared with the scale of  $m_\omega^2$  the extremely narrow  $\Gamma_\omega = 8.4$  MeV allows the isospin violating contribution to be sizeable. Correspondingly the  $\omega \rightarrow 2\pi$  decay must pass through the  $\rho^0$  and thus the attenuation factor is  $\tilde{\Pi}_{\rho\omega}/m_\rho\Gamma_\rho$  and so is down by an order  $\Gamma_\omega/\Gamma_\rho \simeq 0.05$ .

This question of scales and the meaning of the vector meson resonance states themselves must be firmly kept in mind when considering the effects of  $\rho - \omega$  mixing (or indeed any isospin violation). We shall see that this has a recent application.

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With this respect, it is useful to introduce isospin symmetry breaking within a context especially designed in order to account for physics of light vector and pseudoscalar mesons simultaneously and fully. The framework of vector meson dominance (VMD) models is certainly the most appropriate and, among them, the Hidden Local Symmetry (HLS) Lagrangian, with its non-anomalous [14] and anomalous sectors [15] is a good candidate, taking into account its phenomenological success.

In this paper we extend our previous work on symmetry breaking within this context by readdressing first the  $\rho - \omega$  mixing [16,17] and then the full  $\rho - \omega - \phi$  mixing. From our previous studies, we already know how SU(3) symmetry breaking has to be introduced [18] close to lines first proposed in [19]; in order to yield an appropriate description of physics information for decay processes involving  $\eta$  and  $\eta'$  mesons, it has been shown that nonet symmetry in the pseudoscalar (PS) sector should also be broken; a way was proposed in [20] which provides a good understanding of all radiative decays of light mesons. Slightly later [21], we showed that this way of breaking nonet symmetry in the PS sector can be derived from Chiral Perturbation Theory.

However, tree level amplitudes are not sufficient in order to account for the physics of vector mesons. As clear from the observed shape of the pion form factor [22,23], pion loop effects can hardly be neglected when describing the  $\rho$  meson [24–28]. Without a  $\omega - \phi$  mixing mechanism, any description of their decay modes becomes definitely poor. This is traditionally introduced by means of a mixing angle. It has been shown that kaon loop effects are the simplest mechanism within HLS for generating  $\omega - \phi$  mixing [29]. These loop effects can be accommodated within the HLS model in an effective way, by introducing vector meson self-masses and (loop) transition amplitudes like  $\omega \leftrightarrow \phi$  within the Lagrangian. In this way one generates the appropriate pion form factor shape and all corrections to  $\omega/\phi$  decays [29].

However, effects of isospin symmetry breaking (like  $\omega/\phi \rightarrow \pi^+\pi^-$ , for instance) still remain outside the HLS framework. We shall show that the loop mechanism which generates the  $\omega - \phi$  mixing gives a handle to introduce isospin symmetry breaking also by considering loop effects, mainly kaon loops. This mechanism, together with the U(3)/SU(3) symmetry breaking procedure recalled above, will be shown to provide a clear understanding of (almost) all decay modes accessible to a VMD approach.

The outline of the paper is as follows. In Sects. 2, 3 and 4 we define a mechanism for isospin symmetry breaking and examine its consequences on the  $\rho - \omega$  sector in isolation. In Sect. 5, we illustrate with the pion form factor the consistency of this approach; we thus show that the HLS model, broken as we propose, gives to the pion form factor its well known expression in terms of the so-called Orsay phase.

Having shown that this approach is consistent, we extend it in Sect. 6 to a scheme involving the full  $\rho - \omega - \phi$  sector of the HLS Model. We describe in this Section how the coupling constants for all two body decays ( $VPP$ ,  $VP\gamma$ ,

$VVP$ ,  $P\gamma\gamma$ ,  $Ve^+e^-$ ) can be derived from elementary information provided. In this approach, the  $\rho - \omega - \phi$  mixing appears to be the at the origin of the  $\omega/\phi \rightarrow \pi\pi$  decay processes, which are described both in modulus and phase.

In Sect. 7 we apply this model to fit all data related to VMD, except for  $\rho \rightarrow \pi\pi$  and  $K^* \rightarrow K\pi$  which settle a specific problem, not examined here. The picture obtained is impressively successful. Finally, we conclude in Sect. 8. We give in the Appendix most formulae and in three Tables most of our results which cover 26 physics quantities simultaneously fitted within a unified framework.

## 2 Physical fields or ideal fields

In this Section and in the following one, we concentrate on the  $\rho - \omega$  mixing in isolation. This allows to outline the method we use in order to construct a full mixing scheme for vector mesons.

Within a context of an effective field model where we definitely stand, one can assume without any loss of generality that the  $\rho - \omega$  mixing is produced by an effective Lagrangian term of the form

$$\mathcal{L}_{mixing} = \Pi_{\omega\rho}(s)\rho_I\omega_I \quad (2)$$

The use of such a term for parametrising  $\rho - \omega$  mixing is a usual assumption (for a review see [24,25], where a thorough discussion of the origin, properties and values of  $\Pi_{\omega\rho}(s)$  can also be found). Its origin within the HLS model is related to kaon loop effects which introduce a correction of order  $\mathcal{O}((m_{K^+}^2 - m_{K^0}^2)/m_K^2) \sim (m_u - m_d)/m_s$ , as will be discussed in Sect. 4 below.

We have denoted above  $\rho_I$  and  $\omega_I$  the ideal field combinations (*i.e.*, non-strange pure isospin states); the corresponding physical fields will be denoted  $\rho$  and  $\omega$ . For the present purpose, and following general ideas [24,25], we only need to assume that  $\Pi_{\omega\rho}(s)$  is a real analytic function of  $s$  (*i.e.*  $\Pi_{\omega\rho}^*(s) = \Pi_{\omega\rho}(s^*)$ ) where the symbol  $*$  denotes the usual complex conjugation).

Whatever its origin, the term in (2) plays by modifying the  $\rho - \omega$  mass term by adding a non-diagonal piece, as for the kaon loop effects responsible for the  $\omega - \phi$  mixing [29]. Among the possible origins of the term in (2), pion loop effects have been considered. Although this specific contribution is ruled out within the HLS model [30] – which rather privileges kaon loop effects – it has been studied in bi-local field models [31,33]. Following the Renard argument [16], the expected sizeable contribution to the imaginary part of  $\Pi_{\rho\omega}$  from the pion loop is cancelled by a direct  $\omega \rightarrow 2\pi$  term.

### 2.1 Loop effects and mixing

There is a consistent way to account for leading loop effects within the HLS context; this turns out to modify the mass term in the Lagrangian by including all vector meson self-energies and transition amplitudes like  $\phi_I \rightarrow \omega_I$ ,

as discussed in [29], but also<sup>1</sup>  $\rho_I \rightarrow \omega_I$ . In this approach, loop effects are considered by their effects at tree level only through modified vector meson masses.

In this case, the relevant piece of the effective Lagrangian, quadratic in the fields, is given by:

$$\mathcal{L} = \frac{1}{2} \{ [m^2 + \Pi_\rho(s)] \rho_I^2 + [m^2 + \Pi_\omega(s)] \omega_I^2 + 2\Pi_{\omega\rho}(s) \rho_I \omega_I \} \quad (3)$$

Following the approach developed for the  $\omega - \phi$  mixing [29], we have introduced the  $\rho$  and  $\omega$  self-energies as given in [29]; these are real analytic functions of  $s$ . We have also assumed that  $\rho_I$  and  $\omega_I$  have the same (Higgs-Kibble, HK) mass  $m$ ; one could depart from this by starting with a  $SU(2) \times U(1)$  symmetry [32] instead of  $SU(3)$  and thus break the  $\rho - \omega$  mass generacy already at tree level. However, we have preferred here neglecting isospin breaking effects on HK masses, as it is a side issue for the problem under study. Within the HLS model, the common  $\rho - \omega$  mass is [14]  $m^2 = af_\pi^2 g^2$  in terms of the HLS parameter  $a$ , of the universal vector coupling  $g$  and the pion decay constant  $f_\pi$ .

The diagonalization procedure of the HLS Lagrangian with one loop corrections is presented in detail in [29]. We simply recall that the desired diagonalization is obtained by performing the following linear field transformation:

$$\begin{pmatrix} \rho \\ \omega \end{pmatrix} = \begin{pmatrix} \cos \delta(s) & \sin \delta(s) \\ -\sin \delta(s) & \cos \delta(s) \end{pmatrix} \begin{pmatrix} \rho^I \\ \omega^I \end{pmatrix} \quad (4)$$

which connects the physical fields to their ideal combinations. It leads to physical fields which behave like analytic functions of  $s$ ; this can be interpreted as a non-local effect which could be expected as we are not dealing with fundamental (quark and gluon) degrees of freedom. Additionally, it implies at tree level (in this approach) analytical shapes which fit well with physics observations; for instance, the broad shape of the  $\rho$  meson propagator which shows up in the pion form factor is generated here mostly by the pion loop aggregated to the  $\rho_I$  mass term (see (3)).

The angle  $\delta(s)$  – possibly complex – should be chosen in such a way that the mixed  $\rho\omega$  term, appearing still in (3) after the change to physical fields, identically vanishes. This provides<sup>2</sup>:

$$\tan 2\delta(s) = \frac{2\Pi_{\omega\rho}(s)}{\Pi_\rho(s) - \Pi_\omega(s)} \quad (5)$$

and does not depend on the difference of  $\rho_I$  and  $\omega_I$  HK masses, as this vanishes identically in the approximation where we stand. Moreover, the  $s$  dependence of the  $\rho - \omega$  mixing exhibited by (5) is a property already considered [24, 25, 37]. This  $s$  dependence should be expected, as the

<sup>1</sup> At the same order, one might have to include also the parent  $\rho_I \rightarrow \phi_I$  transition amplitude, as will be seen in Sect. 4

<sup>2</sup> It should be noted that the denominator in (5) is nothing but the difference of the  $\rho_I$  and  $\omega_I$  effective running masses as they occur in the Lagrangian (3)

mixing function  $\Pi_{\omega\rho}(s)$  should vanish at  $s = 0$  [17] in any model where the vector mesons couple to conserved currents.

## 2.2 Analytic properties of the angle $\delta(s)$

As noted in [29] for the purpose of  $\omega - \phi$  mixing in isolation, angles like  $\delta(s)$  above are not real for any real  $s$ . In fact, as is clear from (5),  $\sin \delta(s)$ ,  $\cos \delta(s)$  are real analytic functions<sup>3</sup> of  $s$  in an analyticity domain with the same branch point singularities as the various self-energies or transition amplitudes; additional algebraic branch points may occur at odd order zeros or poles of the expression in (5). For our purpose, one only needs to make weak assumptions which ensure that (4) can be inverted as an analytic matrix function: we assume that the analyticity domain contains the upper and lower lips of the physical region  $\{s > 4m_\pi^2\}$  and that both lips can be connected with each other by a continuous path while staying inside this domain. This implies that a segment of  $\{s < 4m_\pi^2\}$  should also belong to this analyticity domain.

The function  $\delta(s)$ , itself, can have logarithmic singularities; however, it never appears as such in the expressions we have to handle.

## 2.3 Interaction terms and self-energies

For completeness, and in order to fix notations for coupling constants, let us recall the relevant interaction piece of the HLS Lagrangian as given in [29] in terms of renormalized fields:

$$\begin{aligned} \mathcal{T}_1 = & -\frac{ia g}{4} Z [\omega_I + \sqrt{2}\ell_V \phi_I] \left[ K^+ \overset{\leftrightarrow}{\partial} K^- + K^0 \overset{\leftrightarrow}{\partial} \bar{K}^0 \right] \\ & -\frac{ia g}{4} \rho_I \left[ Z \left( K^+ \overset{\leftrightarrow}{\partial} K^- - K^0 \overset{\leftrightarrow}{\partial} \bar{K}^0 \right) + 2\pi^+ \overset{\leftrightarrow}{\partial} \pi^- \right] \end{aligned} \quad (6)$$

We do not introduce here loop corrections to vertices, as there is no compelling evidence in favor of observable effects of these with the present data accuracy, even for the  $e^+e^- \rightarrow \pi^+\pi^-$  cross section [26] known nowadays with very good accuracy over a wide range of invariant mass [22, 23].

This Lagrangian piece depends on two  $SU(3)$  breaking parameters, generated by the BKY breaking mechanism [19, 18]  $Z = [f_\pi/f_K]^2 \simeq 2/3$  and  $\ell_V$  fit as  $\simeq 1.4$  (see [20, 29]).

Up to anomalous contributions we neglect (they were estimated negligible in [29]), the  $\rho_I$ ,  $\omega_I$  and  $\phi_I$  self-energies are:

$$\begin{aligned} \Pi_\rho(s) &= 2g_{\rho_I K \bar{K}}^2 \Pi(s) + g_{\rho_I \pi \pi}^2 \Pi'(s) \\ \Pi_\omega(s) &= 2g_{\omega_I K \bar{K}}^2 \Pi(s) \\ \Pi_\phi(s) &= 2g_{\phi_I K \bar{K}}^2 \Pi(s) \end{aligned} \quad (7)$$

<sup>3</sup> Actually, they might only be meromorphic in the physical sheet, as  $\Pi_\rho(s) - \Pi_\omega(s)$  might have zeros in the physical sheet, at  $s = 0$  for instance

(see (D1) and (D4) in [29]) in terms of  $\Pi(s)$  and  $\Pi'(s)$ , respectively the generic kaon and pion loops [29], *i.e.* the loops amputated from their coupling constants to vector mesons. The coupling constants can be read off (6) and obviously fulfill  $|g_{\rho_I K \bar{K}}| = |g_{\omega_I K \bar{K}}|$ .

Let us also recall that the Lagrangian derived after the change of fields fulfills the condition of hermitian analyticity, as in the case of the  $\omega - \phi$  mixing in isolation [29].

### 3 The $\rho - \omega$ mixing “angle”

Using (7), (5) becomes

$$\tan 2\delta(s) = \frac{2\Pi_{\omega\rho}(s)}{g_{\rho_I\pi\pi}^2 \Pi'(s)} \quad (8)$$

It is quite an interesting feature of the HLS model that the difference between the  $\omega_I$  and  $\rho_I$  self-energies is the pion loop to which only  $\rho_I$  couples.

In order to estimate this denominator in the  $\rho - \omega$  peak mass region, one should also keep in mind that this is practically the difference of the (complex and  $s$ -dependent)  $\rho$  and  $\omega$  square masses as they occur in the one-loop corrected Lagrangian. As the width of the  $\omega$  meson is negligible compared to those of the  $\rho$  meson, this gives an input for the real part of the pion loop in (8) valid in the neighborhood of the  $\rho - \omega$  peak. Indeed, in this region, the real part can be identified with the difference of the (Breit–Wigner) masses squared as given in the RPP [34]. Indeed, these are defined as the energy point where the real part of the corresponding propagators goes to zero, or equivalently where the phase goes through  $\pi/2$ . Therefore, writing:

$$g_{\rho_I\pi\pi}^2 \Pi'(s) = R(s) - iI(s), \quad (9)$$

(see (A8) in [29]), we have *locally*, using the RPP masses<sup>4</sup>:

$$R(s \simeq m_\rho^2) \simeq m_\rho^2 - m_\omega^2 \simeq -(1.1 \div 1.9) 10^{-2} \text{GeV}^2 \quad (10)$$

depending on the definition used for the *observed*  $\rho$  mass [34]; this has to be compared with the imaginary part

$$I(s \simeq m_\rho^2) \simeq m_\rho \Gamma_\rho = 0.12 \text{ GeV}^2 \quad (11)$$

Therefore, in the mass region of the  $\rho$  and  $\omega$  mesons, the real part of  $\Pi'(s)$  is negligible and then the denominator in (5) locally reduces to its imaginary part with a good approximation; additionally,  $I(s)$  is positive there, as can be inferred from its explicit expression [29].

<sup>4</sup> Actually,  $R(s)$  is a function of  $s$  which contains logarithms and a subtraction polynomial [28], minimally of the form  $\lambda s$ , with  $\lambda$  real to be fixed by means of appropriate renormalization conditions. The Breit–Wigner formulation turns out to approximate locally this real part by a constant which corresponds to the mass given in the RPP [34]. We call this mass definition *observed* mass and keep in mind that it may have little to do with the masses as they occur in Lagrangians

Within the HLS model,  $\Pi_{\omega\rho}(s)$  arises naturally as the *difference* of both (neutral and charged) kaon loops [30]. These are perfectly defined analytic functions [29] of  $s$  and each contains a subtraction polynomial which should be different for neutral and charged kaon loops, at least in order to account for isospin symmetry breaking for pseudoscalar mesons. This issue will be discussed in some more detail in the next Section, but for the present purpose it is enough to remark that, even neglecting isospin breaking effects on masses (then, some logarithm functions cancel out identically), the HLS expression for  $\Pi_{\omega\rho}(s)$  is essentially a *real valued subtraction polynomial* which has to be determined through renormalization conditions [29]. Standard renormalization conditions [29] imply that this polynomial is minimally of the form  $c \cdot s$ , with real  $c$ , in agreement with general considerations [24,25,17].

Thus, within the HLS model, the numerator is essentially real, and the denominator is largely dominated by the imaginary part of the pion loop in the mass region of interest. Using the coupling constants which can be read off from (6) above, and writing  $cs$  the amplitude for  $\Pi_{\omega\rho}(s)$  amputated from the coupling constants to  $\omega_I$  and  $\rho_I$ , we have:

$$\tan 2\delta(s) \simeq i \frac{Z^2}{2} \frac{cs}{I(s)} \quad (12)$$

Therefore, in contrast with the customary mixing case of  $\omega - \phi$ , the mixing angle is close to being purely imaginary in the mass region of interest (the  $\rho - \omega$  peak value).

The mixing scheme presented here is not in contradiction with more standard formulations in terms of a perturbation parameter (see [24] for instance, or more recently [39]). However, writing it as a complex angle makes the connection with the  $\omega - \phi$  mixing more transparent. Indeed, the nature – real or complex – of these mixing angles follows from peculiarities which could look like kinematical accidents, essentially the relative values of meson masses which determine the  $s$ -regions where the imaginary part of loops are non-zero.

Using the RPP world average mass and width values for the  $\rho$  and  $\omega$  mesons, a better local approximation than (12) for the mixing “angle” can be written

$$\begin{aligned} \tan 2\delta(s \simeq m_\rho^2) \\ = d m_\rho^2 \exp \left\{ i \left[ \pi - \arctan \frac{m_\rho \Gamma_\rho - m_\omega \Gamma_\omega}{m_\omega^2 - m_\rho^2} \right] \right\} \end{aligned} \quad (13)$$

where  $d$  is a real constant to be fit. The explicit phase (referred to below as  $\varphi$ ) becomes<sup>5</sup>:

$$\varphi = \text{Arg}[\tan 2\delta(s \simeq m_\rho^2)] = 100.7 \pm 0.7 \text{ degrees} \quad (14)$$

and does not account for a possible negative sign in the fit value for  $d$ . We will see shortly that this value has to

<sup>5</sup> If we use for  $\rho$  parameters the values given in the entry “ $\tau e^+ e^-$ ” of the RPP [34], instead of the world average which is somewhat less secure, this angle value becomes  $95.5 \pm 0.8$  degrees. Therefore a non-negligible systematic error ( $5^\circ$ , about  $7\sigma_{stat}$ ) can affect this number

be compared with the so-called Orsay phase frequently fit within the pion form factor in the timelike region; among recent fit values, let us quote the value obtained within the HLS framework [26]  $104.7^\circ \pm 4.1^\circ$ .

## 4 SU(2) breaking within the HLS Model

A priori, a straightforward way to introduce isospin symmetry breaking within the HLS model could be through the BKY mechanism [18,19] proved successful when analyzing SU(3) symmetry breaking effects for radiative decays of light mesons [35,20] or the properties of the  $\eta - \eta'$  system [21]. Actually, such an attempt has been already considered in the context of radiative decays of light mesons [36].

Another solution is naturally proposed by the HLS model in close correspondence with the  $\omega - \phi$  mixing. Let us name provisionally  $\ell(K^+K^-)$  and  $\ell(K^0\bar{K}^0)$  the kaon loops amputated from coupling constants to external vector meson lines. These loops are given by dispersion relations [29] which should be subtracted minimally twice in order that the dispersion integrals converge.

This gives rise in both cases to a first degree polynomial in  $s$  with real coefficients in order to satisfy usual analyticity properties (this is discussed in some detail in Appendix A of [29]); let us denote by  $P_\pm(s)$  and  $P_0(s)$  resp., the subtraction polynomials associated with the kaon loops just referred to above. Their coefficients are a priori arbitrary and should be fixed by means of appropriate renormalization conditions. The constant term is always chosen to vanish in theories where vector mesons couple to conserved currents [17] as in the HLS model; the same effect ensures the masslessness of the photon. If isospin is conserved, the first degree terms of both polynomials should clearly be equal. However, if SU(2) is broken, there is no longer any reason for this requirement to be made.

Therefore, breaking of SU(2) symmetry can be implemented by having different renormalization conditions for  $P_\pm(s) = c_\pm s$  and  $P_0(s) = c_0 s$ . Allowing  $c_\pm \neq c_0$  appears to be a consistent effective way to break isospin symmetry within the HLS model at one-loop order.

For clarity, let us denote by  $\ell(K^+K^-) + P_\pm(s)$  and  $\ell(K^0\bar{K}^0) + P_0(s)$ , the full kaon loops, exhibiting this way the (free) subtraction pieces. Up to inessential coefficients related with vector coupling constants and SU(3) breaking effects, we have [30]:

$$\begin{cases} \Pi_{\phi_I\omega_I}(s) \simeq \ell(K^+K^-) + \ell(K^0\bar{K}^0) + P_\pm(s) + P_0(s) \\ \Pi_{\rho_I\omega_I}(s) \simeq \ell(K^+K^-) - \ell(K^0\bar{K}^0) + P_\pm(s) - P_0(s) \\ \Pi_{\rho_I\phi_I}(s) \simeq \ell(K^+K^-) - \ell(K^0\bar{K}^0) + P_\pm(s) - P_0(s) \end{cases} \quad (15)$$

Then, quite generally, the HLS model at one loop allows for transition among the ideal combinations of *all three* neutral vector mesons. It should be remarked that these transitions are associated with kaon loops rather

than with the pion loop<sup>6</sup>. It is also interesting to note that, even if one neglects the  $K^\pm - K^0$  mass difference, the transition amplitudes  $\Pi_{\rho_I\omega_I}(s)$  and  $\Pi_{\rho_I\phi_I}(s)$  do not drop out, even if their imaginary parts identically vanish. Moreover, as loops are analytic functions of  $s$ , real for real  $s$  smaller than the loop threshold, the transition amplitude  $\Pi_{\rho_I\omega_I}(s)$  is certainly real in the region of the  $\rho - \omega$  peak (up to anomalous loop effects). Additionally, it certainly fulfills  $\Pi_{\rho_I\omega_I}(s=0) = 0$ . The order of magnitude of  $\Pi_{\rho_I\omega_I}(s)$  and  $\Pi_{\rho_I\phi_I}(s)$  can be derived from their imaginary parts. By expanding these expressions in the neighborhood of the  $\omega/\phi$  masses, the dominant term can be written as  $[m_{K^+}^2 - m_{K^0}^2]/m_K^2 (= [m_u - m_d]/m_s)$  corrected by a  $3/2 m_K^2/m_{\omega/\phi}^2$  factor.

Therefore, the HLS model allows to have naturally a quasi real  $\Pi_{\rho_I\omega_I}(s)$  in the  $\omega - \rho$  peak region, as obtained from fits [24,13,27]. This illustrates that loop effects can be used as the main mechanism in order to break isospin symmetry by allowing different renormalization conditions to different kaon loops. Stated otherwise, isospin symmetry breaking in the pseudoscalar sector already induces corresponding effects in the vector sector.

Moreover, one observes that the HLS model at one loop, predicts that the full mixing pattern concerns all three neutral vector mesons and establishes the  $\rho_I - \phi_I$  mixing as the physics mechanism for the  $\phi \rightarrow 2\pi$  decay.

## 5 The pion form factor

In order to compute the pion form factor in the timelike region, the relevant piece of the interaction Lagrangian, before changing to physical vector fields<sup>7</sup>, is:

$$\begin{aligned} \mathcal{L} = \dots - i \left[ \frac{ag}{2} \rho_I + e \left(1 - \frac{a}{2}\right) A \right] \cdot [\pi^+ \overleftrightarrow{\partial} \pi^-] \\ - ae f_\pi^2 g \left[ \rho_I + \frac{1}{3} \omega_I \right] \cdot A + \dots \end{aligned} \quad (16)$$

where  $A$  is the electromagnetic field,  $e$  the unit electric charge,  $g$  the universal vector coupling constant and  $a$  the intrinsic HLS parameter fit to  $2.35 \div 2.45$  [26,20,29]. This Lagrangian piece is not affected by SU(3)/U(3) symmetry breakdown.

After the change to physical fields given by (4), it is obvious that SU(2) symmetry breaking generates a direct coupling of  $\omega$  to  $\pi^+\pi^-$ . Denoting by  $g_{\rho\pi\pi}^0 = \frac{ag}{2}$  the unbroken coupling of  $\rho_I$  to a pion pair, the coupling constants for physical  $\rho$  and  $\omega$  are:

$$g_{\rho\pi\pi} = g_{\rho\pi\pi}^0 \cos \delta(s), \quad g_{\omega\pi\pi} = -g_{\rho\pi\pi}^0 \sin \delta(s) \quad (17)$$

As  $\delta(s)$  is close to purely imaginary, this leaves the broken  $\rho$  coupling close to real and the generated coupling of  $\omega$  close to purely imaginary<sup>8</sup>.

<sup>6</sup> In order to be complete, we recall that anomalous terms produce loop effects like  $P\gamma$  or  $VP$  loops which contribute to the transition amplitudes; these have been estimated to be numerically small [29]

<sup>7</sup> We still skip in this Section mixing with the  $\phi$  meson

<sup>8</sup> We recall that  $\sin i\alpha = i \sinh \alpha$  and  $\cos i\alpha = \cosh \alpha$

For sake of conciseness, let us also define:

$$f_{\rho\gamma}^0 = a f_{\pi}^2 g, \quad f_{\omega\gamma}^0 = \frac{a f_{\pi}^2 g}{3}, \quad (18)$$

the  $\rho_I$  and  $\omega_I$  couplings to a photon, as they come out of the standard HLS Lagrangian.

Using the Lagrangian piece in (16) reexpressed in terms of physical vector fields, it is an easy matter to compute the pion form factor. Keeping the leading terms in  $\delta(s)$ , this can be written:

$$F_{\pi}(s) = 1 - \frac{a}{2} - \frac{f_{\rho\gamma}^0 g_{\rho\pi\pi}^0}{D_{\rho}(s)} \cos^2 \delta + \frac{f_{\omega\gamma}^0 g_{\rho\pi\pi}^0}{D_{\omega}(s)} \sin \delta \cos \delta \quad (19)$$

where the  $D_{\rho/\omega}(s)$  are the inverse vector meson propagators written  $D_V(s) = s - m_V^2 + im_V \Gamma_V(s)$  in most phenomenological studies, by releasing the analyticity assumption; in the one-loop Lagrangian we use, these would essentially be written  $D_V(s) = s - m^2 - \Pi_V(s)$ , as already obtained and successfully tested by [28] on  $e^+e^- \rightarrow \pi^+\pi^-$  data.

In order to make the correspondence with (13) and (14), and with usual formulae for the pion form factor [26], let us state  $d/2 = -A$  and use  $\varphi$ , the phase in (14). Assuming  $d$  is small enough, we also have  $\tan 2\delta \simeq \sin 2\delta$  and we can approximate the above expression in the neighborhood of the  $\rho - \omega$  peak by:

$$F_{\pi}(s) = 1 - \frac{a}{2} - \frac{f_{\rho\gamma}^0 g_{\rho\pi\pi}^0}{D_{\rho}(s)} - A e^{i\varphi} \frac{f_{\omega\gamma}^0 g_{\rho\pi\pi}^0}{D_{\omega}(s)} \quad (20)$$

This is nothing but the HLS expression of the pion form factor [26] expressed in terms of the so-called Orsay phase, named here  $\varphi$ .

So, isospin breaking expressed in terms of loop effects gives a consistent picture for the pion form factor and reaches the correct Orsay phase value (see (13)). Therefore, an ‘‘imaginary angle’’ occurring when breaking isospin symmetry is what permits to recover a quite standard and traditional formulation for the pion form factor.

It is an interesting feature that the  $\omega - \rho$  mixing, which expresses isospin symmetry violation, appears in correspondence with the  $\omega - \phi$  mixing, produced by the same sort of loop effects. The specific character of the  $\rho - \omega$  mixing is the dominance of the subtraction term, which carries most of the SU(2) symmetry breaking information in our approach.

In order to be complete, one can estimate the modulus of  $\delta$ . In the vicinity of the  $\omega$  meson mass, one has:

$$|\tan 2\delta|^2 \simeq \frac{\Gamma(\omega \rightarrow \pi^+\pi^-)}{\Gamma(\rho \rightarrow \pi^+\pi^-)} = (1.24 \pm 0.17) 10^{-3} \quad (21)$$

which corresponds to a negligible ‘‘angle’’ of about 1 degree times  $i$ . This is indeed very small but quite comparable in magnitude to the (real)  $\omega - \phi$  mixing angle (about 3 degrees).

## 6 The full mixing pattern

It follows from the Sections above that, basically, the mixing pattern exhibited by the HLS model at one loop involves the full triplet  $(\rho_I, \omega_I, \phi_I)$  as soon as isospin symmetry is broken, as it is in real life. In most physics studies of light meson decays it is usual to neglect isospin breaking effects<sup>9</sup>. A noticeable exception is the pion form factor, because of the important  $\rho - \omega$  interference structure and of the  $\rho - \phi$  interference which shows up through the decay mode  $\phi \rightarrow \pi^+\pi^-$  [34,43,44]. An interesting account of the  $\rho - \phi$  mixing can also be found in the recent [39] in connection with the  $\phi\omega\pi$  coupling.

However, from the final remarks in the Section above, one could ask oneself whether accounting only partly for vector meson mixing effects is legitimate. Indeed, we have just seen that the  $\omega_I - \phi_I$  mixing (measured by its – real– angle) and the  $\rho_I - \omega_I$  mixing (measured by its – imaginary– angle) are quite comparable in magnitude.

### 6.1 Diagonalization procedure

When the  $\phi$  field is ‘‘switched on’’, the effective Lagrangian piece quadratic in the fields changes from (3) to:

$$\begin{aligned} \mathcal{L} = \frac{1}{2} \{ & [m^2 + \Pi_{\rho}(s)] \rho_I^2 + [m^2 + \Pi_{\omega}(s)] \omega_I^2 \\ & + [\ell_V m^2 + \Pi_{\phi}(s)] \phi_I^2 + 2\Pi_{\omega_I\rho_I}(s) \rho_I\omega_I \\ & + 2\Pi_{\omega_I\phi_I}(s) \omega_I\phi_I + 2\Pi_{\rho_I\phi_I}(s) \rho_I\phi_I \} . \quad (22) \end{aligned}$$

Self-energies and transition amplitudes have been defined in (7) and (15) respectively.

In order to compute amplitudes involving the *physical*  $\rho, \omega$  and  $\phi$  mesons, (22) should be diagonalized. This gives the physical fields as algebraic expressions in terms of the ideal field combinations  $\rho_I, \omega_I$  and  $\phi_I$ . In these expressions the coefficients of the (linear) relations are analytic functions of  $s$ , which basically depend on three ‘‘angles’’ through relations much more complicated than (4).

One can obviously define three such ‘‘angles’’ corresponding each to the case where one among  $\rho_I, \omega_I$  and  $\phi_I$  is ‘‘switched off’’. As already stated above, these ‘‘angles’’ are actually (analytic) functions of  $s$  and can be real, imaginary or complex depending on the specific  $s$  value along the physical region.

The  $\omega - \phi$  mixing has been studied in isolation in [29] and the corresponding mixing angle has been found real as long as  $s$  is smaller than the two-kaon threshold; practically, this remains true up the  $\phi$  meson mass region. The  $\omega - \rho$  mixing angle has been considered in the previous Sections and has been found close to purely imaginary. The third mixing angle describes mostly the  $\rho - \phi$  mixing and is named  $\gamma$  below; one can easily show that its imaginary part is certainly large in the mass region of vector meson resonances, but a precise estimate of its real part necessitates assumptions on the subtraction polynomials far beyond the scope of the present paper.

<sup>9</sup> See, however, [36] for an attempt to describe radiative decays of light mesons

$$M = \begin{bmatrix} \cos \delta \cos \beta & -\sin \delta \cos \beta & \sin \beta \\ \sin \delta \cos \gamma + \cos \delta \sin \beta \sin \gamma & \cos \delta \cos \gamma - \sin \delta \sin \beta \sin \gamma & -\cos \beta \sin \gamma \\ \sin \delta \sin \gamma - \cos \delta \sin \beta \cos \gamma & \cos \delta \sin \gamma + \sin \delta \sin \beta \cos \gamma & \cos \beta \cos \gamma \end{bmatrix} \quad (23)$$

## 6.2 Transformation from ideal to physical fields

Therefore the general transformation we are interested in is certainly linear and depends on three angles, each a function of  $s$ .

Moreover, relying on the angle functions obtained by switching off one among  $\rho^0$ ,  $\omega$ ,  $\phi$ , it is likely that these angles vary little along the mass range we are interested in<sup>10</sup>. This leads us to approximate these three analytic functions by three constants, over the mass range covered by the light vector and pseudoscalar mesons. This is a somewhat violent assumption and the ability of the model supplied with this constraint to describe experimental data will teach us about its validity.

This being stated, the transformation which allows to define the physical  $\rho$ ,  $\omega$  and  $\phi$  fields in terms of  $\rho_I$ ,  $\omega_I$  and  $\phi_I$  is formally a rotation and the angles are defined by requiring the vanishing of all mixed terms,  $\rho\omega$ ,  $\rho\phi$ ,  $\omega\phi$  in (22) after the change of fields. The transformation is a real rotation – with real angles – when analytically continued below the two-pion threshold.

This rotation matrix can be chosen as the following CKM-like matrix [34] which was also used in order to study a possible glue component coupled to the  $\eta - \eta'$  system [20,21]: (see (23) on top of the page) and we define the requested field transformation by:

$$\begin{bmatrix} \omega \\ \rho \\ \phi \end{bmatrix} = M \begin{bmatrix} \omega_I \\ \rho_I \\ \phi_I \end{bmatrix}, \quad \begin{bmatrix} \omega_I \\ \rho_I \\ \phi_I \end{bmatrix} = \widetilde{M} \begin{bmatrix} \omega \\ \rho \\ \phi \end{bmatrix} \quad (24)$$

One can indeed check that  $M^{-1} = \widetilde{M}$  whether  $\beta$ ,  $\delta$  and  $\gamma$  are real or complex. As stated above, the sine and cosine functions here are defined through their underlying exponential expressions and coincide with the standard ones for real values of their arguments. We recall that trigonometric functions satisfy all their known properties, even for complex values of their arguments.

If the ‘‘angle’’ functions may become complex, one may wonder that we may be violating hermiticity, as one could have rather expected  $M^{-1} = M^\dagger = \widetilde{M}^*$ . This is not true, as can be seen by going a step prior to the approximation by constants. In this case<sup>11</sup>, the relation fulfilled by  $M(s)$  along the physical region can be written, using obvious notation:

$$M(s + i\varepsilon)M^\dagger((s + i\varepsilon)^*) = M(s + i\varepsilon)M^\dagger(s - i\varepsilon) = 1 \quad (25)$$

assuming that the two lips of the physical region can be connected by a path fully contained in the physical sheet

<sup>10</sup> See [29] for the  $\omega - \phi$  mixing case in isolation

<sup>11</sup> We denote by \* the simple complex conjugation of matrices (with no transposition) and variables

and which does not cross any cut; additionally,  $M(s)$  is real below the 2-pion threshold.

The sine and cosine functions defining  $M(s)$  are certainly algebraic functions of the transition amplitudes and self-energies; therefore, they have essentially the same branch point singularities (plus possible additional ones we will not discuss). Therefore,  $M(s)$  is certainly a real analytic function of  $s$  in a domain sketched several times above. Then, we should have along the physical region:

$$M^*(s - i\varepsilon) = M(s + i\varepsilon) \quad (26)$$

which leads to

$$M(s + i\varepsilon)\widetilde{M}(s + i\varepsilon) = 1 \quad (27)$$

as has been inferred from (24). The precise analyticity domain where this is valid is not easy to study in the present case; this is, furthermore, of no consequence for the present study<sup>12</sup>.

## 6.3 Radiative and vector decays of light mesons

The first important data set we shall analyze are the radiative decays of light mesons. The Lagrangian which allows us to derive their coupling constants can be written:

$$\mathcal{L}_{WZW} = K\epsilon^{\mu\nu\rho\sigma}\text{Tr}[\partial_\mu(eQA_\nu + gV_\nu)\partial_\rho(eQA_\sigma + gV_\sigma)P] \quad (28)$$

where  $Q = \text{Diag}(2/3, -1/3, -1/3)$  is the quark charge matrix and  $A$  is the electromagnetic field.  $P$  is the pseudoscalar field matrix and can be found in [20] with the conventions used here. The vector field matrix is repeated here:

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} (\rho_I + \omega_I)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (-\rho_I + \omega_I)/\sqrt{2} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\phi_I \end{pmatrix}. \quad (29)$$

in order to exhibit that the traditional field  $\rho^0$  is actually the ideal isospin 1 field combination, while the physical field associated with the  $\rho^0$  meson is  $\rho$  in (24). The coefficient  $K$  in (28) is [29]  $K = -3/(4\pi^2 f_\pi)$ .

The various  $VP\gamma$  coupling constants can be derived from (28) in a straightforward way; before rotating to physical fields, they are given in the Appendix.

<sup>12</sup> In order that the framework of what follows holds, one has only to *assume* that this analyticity domain contains a band along  $\text{Re}(s) > 4m_\pi^2$  on both sides of the real axis and, connectedly, a part of the semi-axis  $\text{Re}(s) < 4m_\pi^2$ . This working assumption does not look severe

It should be remarked that (28) is an expression for the VMD assumption which connects the usual anomalous Wess–Zumino Lagrangian for  $P\gamma\gamma$  to its VMD partner  $VP\gamma$  through a common normalization factor ( $K$ ). Therefore, the treatment of  $VP\gamma$  and  $P\gamma\gamma$  couplings differs only by specific symmetry breaking effects.

The physical  $\rho P\gamma$ ,  $\omega P\gamma$  and  $\phi P\gamma$  couplings are easily derived using these ideal couplings and the second (24), by collecting all contributions to the same field combination coupling. Fully developed, they are algebraically rather complicated, even if conceptually simple. They can, however, be easily dealt with within a minimization program.

Let us illustrate one case and, for this purpose, write down symbolically a piece of (28):

$$\dots G_{\rho_I\gamma P}[\rho_I AP] + G_{\omega_I\gamma P}[\omega_I AP] + G_{\phi_I\gamma P}[\phi_I AP] \quad (30)$$

As symbolically, one can derive from (23) and (24) three relations:

$$\rho_I = v_{\rho_I}(\omega)\omega + v_{\rho_I}(\rho)\rho + v_{\rho_I}(\phi)\phi \quad (31)$$

and the corresponding ones for  $\omega_I$  and  $\phi_I$  with, correspondingly,  $v_{\omega_I}$  and  $v_{\phi_I}$ . The three vectors just defined are simply the columns in (23). Rewriting (30) using (31) and the two other parent ones, it remains only to collect all terms contributing to  $[\rho AP]$ ,  $[\omega AP]$  and  $[\phi AP]$  in order to get the coupling constants associated with physical vector mesons.

This allows to include all radiative decay modes in our data sample, *i.e.* the possible 15 decay modes presently all measured. Actually, the  $\pi^0\gamma\gamma$  partial width is not used but replaced by the pion decay constant world average value [34]  $f_\pi = 92.42$  MeV.

Beside the radiative  $VP\gamma$  coupling constants, the Lagrangian (28) defines also the  $VVP$  couplings. From a practical point of view, the interesting piece derived from (28) can be written:

$$\mathcal{L}_1 = -\frac{g^2}{8\pi^2 f_\pi} \{ [\omega_I \rho_I \pi^0] + [\omega_I \rho^+ \pi^-] + [\omega_I \rho^- \pi^+] \} \quad (32)$$

using obvious notations. From this, we can derive the  $\phi\omega\pi^0$  coupling which allows to include the corresponding decay mode in our data sample. The coupling constants for  $\phi\rho\pi$  [38] is derived from fits to the  $e^+e^- \rightarrow \pi^+\pi^-\pi^0$  cross section; the most recent fit value [38]  $g_{\phi\rho\pi} = 0.815 \pm 0.021$  GeV<sup>-1</sup> is seemingly well established. Its parent  $\omega\rho\pi$  is subject to more controversy [39–42] and the reported values range between  $11.7 \pm 0.5$  GeV<sup>-1</sup> [39] for the smallest to  $16.1 \pm 0.4$  GeV<sup>-1</sup> for the largest, with a preferred value [40] around  $14.3$  GeV<sup>-1</sup>; until clarification, it seems more secure to leave this information outside fits and simply compare with our predictions.

Finally the relative phase of the coupling constants  $\phi\omega\pi^0$  and  $\omega\rho\pi$  comes from a fit to  $e^+e^- \rightarrow \omega\pi^0$ ; the most recent estimate [42] is  $-49^\circ \pm 7^\circ \pm 1^\circ$  and will be included into the data sample we shall fit.

## 6.4 Information from $e^+e^-$ , $\pi^+\pi^-$ and $K\bar{K}$ decays

Concerning the decay of vector mesons to  $e^+e^-$ , the relevant Lagrangian piece is [20]:

$$\mathcal{L}_{em} = -ae f_\pi^2 g \left[ \rho_I + \frac{1}{3}\omega_I + \ell_V \frac{\sqrt{2}}{3}\phi_I \right] \cdot A \quad (33)$$

which depends on the breaking parameter [19,18,20]  $\ell_V$ . It allows to derive the corresponding couplings for the physical fields  $\rho$ ,  $\omega$  and  $\phi$  using (24) above.

The other HLS Lagrangian piece given in (6) provides the coupling constants of the physical  $\phi$  meson to both  $K^+K^-$  and  $K_L K_S$  final state. Finally, the  $\pi^+\pi^-$  term in (6) gives the  $\omega\pi^+\pi^-$  and  $\phi\pi^+\pi^-$  couplings which allows us to include these partial widths inside our data sample.

Therefore, in addition to the 14 modes, the  $\phi \rightarrow \omega\pi^0$  decay width and its phase relative to  $\omega\rho\pi$ , and the  $\phi\rho\pi$  coupling as stated in the section above, we can add 7 more decay modes to our working data sample ( $\rho/\omega/\phi \rightarrow e^+e^-$ ,  $\phi \rightarrow K^+K^-/K_L K_S$ ,  $\omega/\phi \rightarrow \pi^+\pi^-$ ).

As clear from the above Sections dealing with the  $\rho - \omega$  mixing, the phase of the  $\omega$  term (denoted above  $\varphi$ ) relative to the  $\rho$  term carries as much physics information as the  $\omega \rightarrow \pi^+\pi^-$  partial width (one gives the phase of the breaking term, the other its modulus). Referring to the Review of Particle Properties [34], there is no reported average value and the latest fit which could have produced such information did not include this measurement [23], therefore, we shall use the latest published fit value [26]  $104.7^\circ \pm 4.1^\circ$  as reference data.

There are however, former fit values for the Orsay phase which give information on its model dependence [45–47]. The reference value we choose is somewhat median and has the virtue to reproduce the threshold behavior predicted by Chiral Perturbation Theory with a good accuracy [26]; however, from the other references just quoted, one might conclude that systematic errors of about  $10^\circ$  are not unlikely.

On the other hand, the fit of the  $\phi \rightarrow \pi^+\pi^-$  rate has been done several times [34,43,44], but only one value for the phase is currently reported in the literature [43] and provides valuable physics information. The reported value is  $\psi = -34^\circ \pm 4^\circ$ ; this corresponds to a definition where the  $\phi$  inverse propagator is written  $m_\phi^2 - s - im_\phi\Gamma_\phi$ , opposite in sign to the definition currently used (see Sect. 5). In order to recover consistency with the rest of the information we use, a minus sign should be absorbed in this phase which thus becomes  $\psi = 146^\circ \pm 4^\circ$ .

These phases are the phases of the following quantities:

$$F_V = f_{V\gamma} G_{V\pi^+\pi^-} \quad , \quad V = \rho, \omega, \phi \quad (34)$$

which are allowed to be complex for all vector mesons.

Therefore, taking into account these 2 additional phases, our data sample contains 26 physics quantities; indeed, all modes reported above, except for  $\pi^0 \rightarrow \gamma\gamma$  replaced by the world average value for  $f_\pi$ , the controversial coupling  $g_{\omega\rho\pi}$ , to which we shall nevertheless compare, as well as the phase of  $G_{\rho\pi\pi}$  (unavoidably generated



by isospin breaking) which should be (and is found) very small.

## 7 Fitting the data sample

We have fit the set of data listed above within the model presented in the above Section concerning the full mixing pattern and in the Appendix concerning the rest of the parameter set.

Concerning the data, all have been taken from the last issue of the Review of Particle Properties (RPP) [34]<sup>13</sup>. For data which have no existing entry in the RPP, we have chosen the latest reference. Therefore, among all physics quantities which could be accessed by the model we present, only the major modes  $K^* \rightarrow K\pi$  and  $\rho \rightarrow \pi\pi$  are left aside, as already stated.

### 7.1 Analysis of the fit conditions

In this Section, we aim at making clear, which are the parameters we use and the level of freedom allowed by their existing measured value; afterwards, we describe the various fit strategies we have followed.

The unbroken HLS model [14] basically depends on a very few parameters which are not predicted and should be extracted from data. Beside the unit electric charge  $e$  which is certainly well determined, these are the universal vector meson coupling  $g$ , the pion decay constant  $f_\pi = 92.42$  MeV and  $a$ , a dimensionless parameter specific of the HLS model. In standard VMD models, one has  $a = 2$ , however within the HLS model this condition can be relaxed; in this case, fits to experimental data [57, 20, 21, 23] indicate that  $a = 2.3 \div 2.5$  should be preferred. This merely means that a small coupling  $\gamma\pi^+\pi^-$  survives beside vector mesons exchanges.

Concerning symmetry breaking parameters, previous studies [20, 21, 29] have already reduced the fit freedom by relating, and/or fixing the breaking parameters specific to the pseudoscalar sector: we have already  $Z = [f_\pi/f_K]^2 = 2/3$ , as a consequence of  $F_K(0) = 1$  [19, 18]; it has been checked that the set of radiative and leptonic decays favors this value unambiguously [20]. However, in view of the new result [6] on this subject, we shall make a separate study of this parameter.

As all other models, the HLS model requires mixing in the  $\eta/\eta'$  sector, however, the two parameters involved there (the mixing angle  $\theta_P$  and the nonet symmetry breaking parameter  $x$ ) are algebraically related by the HLS phenomenology [21] (see (A1)) with a high accuracy.

In the vector sector of the HLS model, two breaking parameters, denoted here  $\ell_V$  and  $\ell_T$ , seem unavoidable

and free, even if  $\ell_V$  might be fixed sometime, when vector meson masses would be clearly understood<sup>14</sup> [18, 20, 29].

This being stated, there remain 3 complex “angles” (6 parameters) which are the body of the present study. Within some approximations (mostly, neglecting anomalous loops), one has already noted some clear guesses: one should be mostly real (it corresponds to the standard  $\omega - \phi$  mixing angle [29]), another close to purely imaginary (it corresponds to the  $\omega - \rho$  mixing angle examined in Sects. 3–5). Of course, when going to numerical analysis, the validity of these guesses can be controlled. Moreover, the expectations just referred to have been established above or elsewhere [29] by considering mixing patterns in isolation; therefore, slight departures from these expectations are not unlikely.

i/ As first attempt, we have left these 6 parameters free in the fit. We reached the good fit quality of  $\chi^2/\text{dof} = 13.41/15$  (26 data, 11 parameters) which corresponds to a 57% probability. The fit correlation matrix was observed to exhibit large correlations between fit values for  $\beta$  and  $\gamma$  and between  $\text{Re}(\gamma)$  and  $\text{Im}(\gamma)$ ; as this could well influence the fit procedure, we have looked for equivalent parametrizations. The most appropriate we found was to use  $\text{Re}(\beta)$ ,  $\text{Im}(\beta)$ ,  $\text{Im}(\gamma)$  and a parameter  $k$  defined by  $\text{Re}(\gamma) = k \text{Im}(\gamma)$ . In this case we improved the fit quality to  $\chi^2/\text{dof} = 12.59/15$ , corresponding to a fit probability of 63%. The fit then returned  $k = -0.23^{+0.40}_{-0.60}$ , which indicates that the angle  $\gamma$  can be chosen imaginary ( $k = 0$ ); the global fit assuming this constraint is given in the first data column of Table 1.

ii/ We have explored several strategies in order to reduce the freedom in fits by fixing several subsets of parameters. The most interesting results, with reasonable probabilities (above the percent level), are given in the second and third data columns in Table 1. The former relies on the observation that the  $\omega - \phi$  mixing angle is well fitted real [29]; this leads to try requesting  $\text{Im}[\beta] = 0$  in order to lessen the fit freedom. The latter relies on the observation that both  $\text{Re}[\gamma]$  and  $\text{Re}[\delta]$ , basically related with the mixing of  $\omega$  and  $\phi$  to  $\rho$ , are quite generally yielded small compared to the corresponding real parts; this is true in the general framework under examination and also in studies where these mixing phenomena were considered in isolation. Even if somewhat brutal, these approximations lead both to quite reasonable fit quality.

iii/ We have also considered that there can be a functional relation between some “angles”. From their expressions in terms of pseudoscalar meson loops, one might guess that the “angles”  $\gamma$  and  $\delta$  could be functionally related. As we are dealing with slowly varying functions over the range of interest, we have tried requesting:

$$\gamma = (\mu_1 + i\mu_2) \delta . \quad (35)$$

<sup>14</sup> We mean by this, that the relation between theoretical masses as they occur in Lagrangians and the corresponding measured quantities is unclear for broad objects like  $\rho$  or  $K^*$ . This problem is certainly related with the apparent difficulty to accommodate the major decay modes of  $K^*$  and  $\rho$  and all other decay modes *simultaneously* within the HLS framework

<sup>13</sup> An update of the Particle Data Table can be found at <http://pdg.lbl.gov>; some minor modifications have been made to the decay rates considered in the present paper. They do not affect our analysis

**Table 1.** Fit results under various strategies. Parameter values written boldface means that they are not allowed to vary; this translates mathematically the fit condition given on the top of the Table

Fixing Angle Properties	$\rho - \phi$ Imaginary only	$\omega - \phi$ Real only	$\omega - \rho$ and $\rho - \phi$ Imaginary	$\omega - \rho$ and $\rho - \phi$ proportional
$g$	$5.651 \pm 0.017$	$5.652 \pm 0.017$	$5.641 \pm 0.017$	$5.652 \pm 0.017$
$\theta_P$ [deg.]	$-10.33 \pm 0.20$	$-10.32 \pm 0.20$	$-10.33 \pm 0.20$	$-10.34 \pm 0.20$
$a$ [HLS]	$2.517 \pm 0.035$	$2.523 \pm 0.034$	$2.485 \pm 0.033$	$2.513 \pm 0.035$
$\ell_V$	$1.343 \pm 0.021$	$1.337 \pm 0.021$	$1.366 \pm 0.021$	$1.346 \pm 0.021$
$\ell_T$	$1.231 \pm 0.052$	$1.230 \pm 0.052$	$1.232 \pm 0.052$	$1.230 \pm 0.052$
Re[ $\beta$ ]	$-0.058 \pm 0.003$	$-0.061 \pm 0.002$	$-0.054 \pm 0.003$	$-0.056 \pm 0.003$
Im[ $\beta$ ]	$-0.020 \pm 0.005$	<b>0.</b>	$-0.028 \pm 0.003$	$-0.029 \pm 0.002$
Re[ $\delta$ ]	$(0.52 \pm 0.18) 10^{-2}$	$(0.54 \pm 0.19) 10^{-2}$	<b>0.</b>	$(0.55 \pm 0.19) 10^{-2}$
Im[ $\delta$ ]	$(-0.29 \pm 0.02) 10^{-1}$	$(-0.29 \pm 0.02) 10^{-1}$	$(-0.31 \pm 0.02) 10^{-1}$	$(-0.29 \pm 0.02) 10^{-1}$
Re[ $\gamma$ ]	<b>0.</b>	$(-0.57 \pm 0.15) 10^{-3}$	<b>0.</b>	$(.031 \pm .005) \text{Re}[\delta]$
Im[ $\gamma$ ]	$(-0.96 \pm 0.18) 10^{-3}$	$(-1.16 \pm 0.16) 10^{-3}$	$(-1.06 \pm 0.18) 10^{-3}$	$(.031 \pm .005) \text{Im}[\delta]$
$\chi^2/\text{dof}$	12.88/16	17.06/16	20.94/17	12.59/16
Probability	63%	38%	23%	70%

It happens that this relation is well accepted by the data. The fit returned  $\mu_1 = (0.030_{-0.006}^{+0.007})$  and  $\mu_2 = (0.011_{-0.114}^{+0.131}) 10^{-1}$  with  $\chi^2/\text{dof} = 12.58/15$  (probability 63%). Therefore, requiring the condition in (35) is certainly justified and additionally, one gets phenomenological motivation to require  $\mu_2 = 0$  from start. The corresponding fit results are displayed in the fourth data column of Table 1. The fit quality reached can hardly be better.

Finally, an additional fit (not shown) assuming  $\text{Im}[\beta] = 0$  and leaving free  $\mu_1$  and  $\mu_2$  has been performed in order to test the stability of other fit parameters, by requiring a condition expected if one interprets  $\beta$  as strictly equivalent to the  $\omega - \phi$  mixing angle in isolation. The result practically coincides with the second data column in Table 1, including its fit quality, and returns  $\mu_1 = (0.354 \pm 0.044) 10^{-1}$  and  $\mu_2 = (-0.270 \pm 0.040) 10^{-1}$ . The various contributions to the  $\chi^2$  implies that this fit and the second data column in Table 1 give the same description of the data with the same probability.

iv/ From the results given in Table 1, it is clear that most parameter values do not depend sensitively on the fit strategy considered. As all fit qualities are especially favorable, no strategy can be privileged. The single parameter which seems floating is  $\text{Re}[\gamma]$  which cannot be better constrained before improving the accuracy of existing information for  $\phi \rightarrow \pi\pi$  in modulus and phase, and/or improving the phase of the  $\phi\omega\pi$  coupling constant. Whether  $\gamma$  could be removed as a whole has been considered with a negative answer. Indeed, performing a fit with  $\gamma = 0$  leads to a quality which becomes really poor ( $\chi^2/\text{dof} = 36.34/17$ , probability 0.4%).

It should be stressed that the information prominently affected by isospin symmetry breaking represents 6 measurements ( $\omega/\phi \rightarrow \pi\pi$ ,  $\phi \rightarrow \omega\pi$  in modulus and phase), which requires in our approach 4 parameters ( $\delta$ ,  $\mu_1$  and

$\mu_2/\text{Im}[\beta]$ ). Therefore, even in this sector, the set of parameters is reasonably constrained and only waits for more accurate data.

## 7.2 Analysis of fit parameter values

As first remark, it is clear that all fit parameters not connected with vector meson mixing, (the five first lines in Table 1), are quite stable and their values compare well with previous attempts along the present lines [20, 29, 21]. We note, however, the correlation between  $\ell_V$  and  $a$  which reaches  $-90\%$ ; this correlation is purely numerical and reflects that the dependence upon  $\ell_V$  within the set of coupling constants is actually a dependence upon the product  $a\ell_V$ .

The value obtained for the pseudoscalar mixing angle has been discussed in [21] and agrees quite well with recent estimates from lattice QCD [48]. It has been shown in [21] that this angle is (algebraically) related with the mixing angle  $\theta_8$  in favor within the ChPT community by a factor which can be predicted close to 2.

As stated above, the mixing ‘‘angle’’  $\beta$  can be considered as intimately associated with  $\omega - \phi$  mixing. It should be noted that the value of  $\text{Re}(\beta)$  varies little when constraints are put on other parameters. This real part is a  $30\sigma$  effect and corresponds to an  $\omega - \phi$  mixing angle of  $-(3.2 \div 3.5)^\circ \pm 0.11^\circ$ , that is smaller than the ideal mixing angle, as found in [20, 29]. These remarks allow to conclude that introducing isospin symmetry breaking, as we propose does not affect sensitively the sector of radiative and leptonic decays. For most parameters not intimately related with isospin symmetry breaking, this follows expectations (see the first five lines in Table 1); however, because of the ‘‘rotation’’ matrix structure, it was not obvious that  $\text{Re}[\beta]$  could not shift by a few degrees, pushing

**Table 2.** Reconstructed Branching fractions for radiative and leptonic decays using the various fit strategies. The last column displays the recommended values from the Review of Particle Properties [34]. The last line gives a reminder of the fit quality given in Table 1

Fixing Angle Properties	$\rho - \phi$ Imaginary only	$\omega - \phi$ Real only	$\omega - \rho$ and $\rho - \phi$ Imaginary	$\omega - \rho$ and $\rho - \phi$ proportional	PDG
$\rho \rightarrow \pi^0 \gamma$ [ $\times 10^4$ ]	$5.36 \pm 0.12$	$5.37 \pm 0.13$	$5.18 \pm 0.10$	$5.37 \pm 0.13$	$6.8 \pm 1.7$
$\rho \rightarrow \pi^\pm \gamma$ [ $\times 10^4$ ]	$5.13 \pm 0.10$	$5.13 \pm 0.10$	$5.10 \pm 0.10$	$5.13 \pm 0.10$	$4.5 \pm 0.5$
$\rho \rightarrow \eta \gamma$ [ $\times 10^4$ ]	$3.18 \pm 0.08$	$3.18 \pm 0.08$	$3.15 \pm 0.08$	$3.18 \pm 0.08$	$2.4_{-0.9}^{+0.8}$
$\eta' \rightarrow \rho \gamma$ [ $\times 10^2$ ]	$33.91 \pm 3.16$	$33.92 \pm 3.13$	$33.52 \pm 3.04$	$33.93 \pm 3.16$	$30.2 \pm 1.3$
$K^{*\pm} \rightarrow K^\pm \gamma$ [ $\times 10^4$ ]	$9.89 \pm 1.01$	$9.78 \pm 1.01$	$9.85 \pm 1.03$	$9.89 \pm 1.01$	$9.9 \pm 0.9$
$K^{*0} \rightarrow K^0 \gamma$ [ $\times 10^3$ ]	$2.31 \pm 0.33$	$2.31 \pm 0.32$	$2.30 \pm 0.32$	$2.31 \pm 0.33$	$2.3 \pm 0.2$
$\omega \rightarrow \pi^0 \gamma$ [ $\times 10^2$ ]	$8.49 \pm 0.10$	$8.49 \pm 0.10$	$8.48 \pm 0.11$	$8.49 \pm 0.10$	$8.5 \pm 0.5$
$\omega \rightarrow \eta \gamma$ [ $\times 10^4$ ]	$7.72 \pm 0.15$	$7.74 \pm 0.16$	$7.86 \pm 0.14$	$7.69 \pm 0.15$	$6.5 \pm 1.0$
$\eta' \rightarrow \omega \gamma$ [ $\times 10^2$ ]	$2.79 \pm 0.26$	$2.77 \pm 0.26$	$2.89 \pm 0.26$	$2.79 \pm 0.26$	$3.03 \pm 0.31$
$\phi \rightarrow \pi^0 \gamma$ [ $\times 10^3$ ]	$1.37 \pm 0.09$	$1.36 \pm 0.09$	$1.38 \pm 0.09$	$1.38 \pm 0.09$	$1.26 \pm 0.10$
$\phi \rightarrow \eta \gamma$ [ $\times 10^2$ ]	$1.29 \pm 0.02$	$1.28 \pm 0.02$	$1.29 \pm 0.02$	$1.29 \pm 0.02$	$1.297 \pm 0.033$
$\phi \rightarrow \eta' \gamma$ [ $\times 10^4$ ]	$0.58 \pm 0.02$	$0.59 \pm 0.02$	$0.58 \pm 0.02$	$0.58 \pm 0.02$	$0.67_{-0.31}^{+0.35}$
$\eta \rightarrow \gamma \gamma$ [ $\times 10^2$ ]	$39.45 \pm 3.74$	$39.43 \pm 3.74$	$39.32 \pm 4.02$	$39.45 \pm 3.74$	$39.33 \pm 0.25$
$\eta' \rightarrow \gamma \gamma$ [ $\times 10^2$ ]	$2.13 \pm 0.20$	$2.13 \pm 0.20$	$2.18 \pm 0.19$	$2.13 \pm 0.20$	$2.12 \pm 0.14$
$\rho \rightarrow e^+ e^-$ [ $\times 10^5$ ]	$4.70 \pm 0.16$	$4.73 \pm 0.16$	$4.54 \pm 0.15$	$4.69 \pm 0.16$	$4.49 \pm 0.22$
$\omega \rightarrow e^+ e^-$ [ $\times 10^5$ ]	$6.96 \pm 0.21$	$6.94 \pm 0.21$	$7.06 \pm 0.22$	$6.97 \pm 0.21$	$7.07 \pm 0.19$
$\phi \rightarrow e^+ e^-$ [ $\times 10^4$ ]	$2.96 \pm 0.04$	$2.96 \pm 0.04$	$2.96 \pm 0.04$	$2.96 \pm 0.04$	$2.91 \pm 0.07$
$\chi^2/\text{dof}$	12.88/16	17.06/16	20.94/17	12.59/16	
Probability	63%	38%	23%	70%	

the  $\omega - \phi$  mixing angle slightly above its ideal value. This is not observed, whatever the fit strategy.

All uncertainties in the fits are connected mainly with the values for  $\text{Im}(\beta)$ ,  $\text{Re}(\gamma)$  and  $\text{Re}(\delta)$ . This reflects that, even though valuable, most isospin breaking data are still of rather poor accuracy.

### 7.3 Reconstruction of physics quantities

The fit parameter values allow to reconstruct branching fractions, coupling constants and phase factors as predicted by our model. Dealing with errors is done by Monte Carlo methods using the full covariance matrix of each fit in order to account properly for correlations. Let us denote  $V_{ij}$  the covariance matrix element for parameters  $x_i$  and  $x_j$ , by  $\lambda_\alpha$  its eigenvalues and by  $a_i^\alpha$  the  $i^{\text{th}}$  component of the  $\alpha^{\text{th}}$  normalized eigenvector; then any measured parameter  $x_i$  can be considered as a random variable given by:

$$x_i = x_i^0 + \sum_{\alpha=1}^n \varepsilon^\alpha \sqrt{\lambda_\alpha} a_i^\alpha \quad (36)$$

where  $x_i^0$  is the central value returned by the fit and  $\{\varepsilon^\alpha, \alpha = 1, \dots, n\}$  is a set of independent gaussian random variables of zero mean and unit standard deviation ( $\langle \varepsilon^\alpha \varepsilon^\beta \rangle = \delta_{\alpha\beta}$ ).

The fit quantities were the coupling constants for each process. These have been derived from the accepted branching fractions [34] – taking into account their accuracy – and assuming that the full widths and masses of mesons are random variables.

In order to reconstruct the physics (measured) quantities, in addition to considering the fit parameters as correlated random variables, we have assumed the mass and width of each vector meson as independent random variables with standard deviation given by the accepted errors [34]; instead, all masses of pseudoscalar mesons were considered as fixed, except for the  $\eta'$  meson. Finally, for the  $\rho$  meson (charged and neutral) we have considered the value given in the  $\tau/e^+e^-$  entry of the RPP [34] for its mass and width. We thus follow the conclusion of the ALEPH Collaboration who saw no difference for these parameters – within errors – between the charged and neutral modes [50].

#### 7.3.1 Radiative and leptonic decay modes

We give in Table 2 the reconstructed branching fractions for radiative and leptonic decays together with the recommended values [34].

It is interesting to compare these reconstructed values with previous fits done using the model we present, without introducing isospin symmetry breaking (see Table III

and IV in [20] and Table 2 in [21], where nonet symmetry breaking and the pseudoscalar mixing angle have been algebraically related). All changes are actually tiny, confirming that breaking of isospin symmetry contributes little in this realm. However, two small changes can be noticed.

The first is that  $\text{BR}(\rho^0 \rightarrow \pi^0\gamma)$  becomes larger than  $\text{BR}(\rho^\pm \rightarrow \pi^\pm\gamma)$  by 4.7% and the predicted branching fraction  $\text{BR}(\phi \rightarrow \pi^0\gamma)$  increases by 8%. These are clearly consequences of breaking isospin symmetry. Otherwise, whatever the additional conditions stated, the general agreement of the reconstructed physics quantities with the data collected and averaged in the RPP can hardly be better.

Among the recent changes in the RPP, one should notice the branching fraction for  $\phi \rightarrow \eta'\gamma$  which has now a central value in much better agreement with our model prediction. On the other hand, some new measurements have been recently reported which have not influenced the RPP recommended values (neither our fits) and might be commented.

First, the new measurement  $\text{BR}(\rho^0 \rightarrow e^+e^-) = (4.67 \pm 0.15) 10^{-5}$  reported by CMD-2 [51] remains in good agreement with our fit values. The second new measurement [52]  $\text{BR}(\rho \rightarrow \eta\gamma) = (3.28 \pm 0.36 \pm 0.24) 10^{-4}$  has a higher central value in better agreement with our reconstructed value, as for  $\text{BR}(\phi \rightarrow \eta\gamma) = (1.287 \pm 0.013 \pm 0.063) 10^{-2}$ . The third new measurement [52]  $\text{BR}(\omega \rightarrow \eta\gamma) = (5.10 \pm 0.72 \pm 0.34) 10^{-4}$  is in relatively poorer agreement with our predictions than the RPP mean value [34].

The SND Collaboration has also published new results on  $\eta\gamma$  decays of vector mesons [53–55]; the branching fractions reported are in good correspondence with our predictions. However, as for the CMD-2 result reported above, the new SND data for  $\text{BR}(\omega \rightarrow \eta\gamma) = (4.62 \pm 0.71 \pm 0.18) 10^{-4}$  might indicate that our prediction for this mode is slightly too large.

As the predictions for  $\text{BR}(\omega \rightarrow \eta\gamma)$  are alike whatever the conditions on the model, this (possible)  $2\sigma$  disagreement could point towards a mass dependence of the mixing “angles”.

Before closing this Section, it is of relevance to comment on a recent claim [56] that isospin symmetry breaking might be much larger in  $\rho^0 \rightarrow \pi^0\gamma$  than anywhere else. From what has just been commented, it is clear that a  $\simeq 5\%$  effect of isospin symmetry breaking is well accepted by all data, the former [34, 57] and the recent SND datum as well ( $\text{BR}(\rho^0 \rightarrow \pi^0\gamma) = (4.3 \pm 2.2 \pm 0.04) 10^{-2}$ ) [58]. We have checked that the central value claimed by [56] (about a factor of 2 in rates) cannot be reproduced in consistency with the rest of radiative decays.

### 7.3.2 $\pi\pi$ decay modes

For the  $\phi$  and  $\omega$  decays to  $\pi\pi$ , we have used the recommended branching fractions [34] and the phases fit resp. by [43] and [26]. Table 2 shows that they are well reproduced by any of our fits. Taking into account the uncertainties already quoted for the Orsay phase, we even cannot rule out the solution given by the third data column.

The Collaboration CMD-2 has recently provided [23]  $\text{BR}(\omega \rightarrow \pi\pi) = (1.32 \pm 0.23)\%$  significantly smaller than the recommended value  $(2.21 \pm 0.30)\%$  we have used, and  $\text{BR}(\phi \rightarrow \pi\pi) = (1.60 \pm 0.49) 10^{-4}$  about  $2\sigma$  larger than the RPP value  $(0.75 \pm 0.14) 10^{-4}$ . No phase measurement has been correspondingly reported.

It is worth commenting on the possible effects of these new measurements. These have been examined within the framework of our preferred fit strategy (the one reported in the fourth data column of Table 1).

We have first changed  $\text{BR}(\omega \rightarrow \pi\pi)$  to the new CMD-2 datum. The best fit obtained provides  $\chi^2/\text{dof} = 13.68/16$  (probability 62%). The parameter values and errors are the same as in the fit reported in Table 1, except that  $\text{Im}\delta$  yields a reduced magnitude ( $-0.029 \pm 0.002$  becomes  $-0.023 \pm 0.002$ ); on the other, hand  $\mu_1$  changes from  $0.031 \pm 0.005$  to  $0.020 \pm 0.007$ . Finally, the contribution of the Orsay phase to the global  $\chi^2$  is about 0.12 and does not change, showing that the datum used remains consistent with the rest.

Having restored  $\text{BR}(\omega \rightarrow \pi\pi)$  to the RPP recommended value, we have changed the datum for  $\text{BR}(\phi \rightarrow \pi\pi)$  to the new result of CMD-2. The single significant change with respect to Table 1 is the value of  $\mu_1$  ( $0.031 \pm 0.005$  becomes  $0.017 \pm 0.001$ ) and the fit returned  $\chi^2/\text{dof} = 13.44/16$  (probability 64%).

Finally performing both changes simultaneously provides a fit with  $\chi^2/\text{dof} = 14.83/16$  (probability 54%) with results merging the changes mentioned just above.

Therefore, even if some uncertainty remains for the values of the  $\gamma$  and  $\delta$  angles, the model exhibits enough flexibility in order to accommodate significant changes in some crucial data. Actually, the two modes just commented determine almost solely the magnitude of isospin symmetry breaking.

It should also be noted that the changes just mentioned in the branching fractions do not give rise to inconsistencies with the phases of the corresponding coupling constants we have used, which thus look more firmly established.

### 7.3.3 $VVP$ couplings and $3\pi$ decays

In all fit strategies and even by changing to new data as reported just above, the information concerning the  $VVP$  processes is remarkably stable.

One should thus note the nice agreement with the data reported by the SND Collaboration on the  $\phi \rightarrow \omega\pi^0$  process [39, 42] both in branching fraction and phase.

The SND datum [38] for  $|g_{\phi\rho\pi}|$  is also reproduced with good accuracy. As the phase of this coupling constant is unfortunately not reported we have no reference datum to which our prediction could be compared. Such information is in principle accessible from fit to  $e^+e^- \rightarrow \pi^+\pi^-\pi^0$  data [38], but the existence of a (complex) non resonant term<sup>15</sup> in the amplitude renders this extraction hasardous.

<sup>15</sup> This term might account for the box anomaly, but also for high mass resonances and this last effect seems hard to model in both modulus and phase

**Table 3.** Reconstructed Branching fractions from various fit strategies, cont'd. The last column displays the recommended values from the Review of Particle Properties [34]. The last line reminds the global fit quality given in Table 1. The datum indicated by (\*) has been corrected in order to absorb a minus sign (see text)

Fixing Angle Properties	$\rho - \phi$ Imaginary only	$\omega - \phi$ Real only	$\omega - \rho$ and $\rho - \phi$ Imaginary	$\omega - \rho$ and $\rho - \phi$ proportional	PDG Reference
$\phi \rightarrow K^+ K^- [\times 10^2]$	$50.25 \pm 0.72$	$50.26 \pm 0.71$	$50.24 \pm 0.72$	$50.22 \pm 0.73$	$49.2 \pm 0.7$
$\phi \rightarrow K_S^0 K_L^0 [\times 10^2]$	$32.95 \pm 0.47$	$32.95 \pm 0.47$	$32.94 \pm 0.48$	$32.92 \pm 0.48$	$33.8 \pm 0.6$
$\omega \rightarrow \pi^+ \pi^- [\times 10^2]$	$2.23 \pm 0.30$	$2.19 \pm 0.29$	$2.32 \pm 0.31$	$2.26 \pm 0.30$	$2.21 \pm 0.30$
phase of $g_{\omega\pi^+\pi^-}$ [degr]	$103.50 \pm 4.02$	$106.3 \pm 3.81$	$92.34 \pm 0.69$	$103.40 \pm 3.88$	$104.7 \pm 4.1$
$\phi \rightarrow \pi^+ \pi^- [\times 10^5]$	$7.93 \pm 1.40$	$8.15 \pm 1.45$	$7.60 \pm 1.24$	$7.70 \pm 1.43$	$7.5 \pm 1.4$
phase of $g_{\phi\pi^+\pi^-}$ [degr]	$146.30 \pm 3.95$	$147.5 \pm 4.09$	$146.0 \pm 3.75$	$145.95 \pm 3.93$	$146.0 \pm 4.0(*)$
$\phi \rightarrow \omega\pi^0 [\times 10^5]$	$4.10 \pm 0.48$	$3.62 \pm 0.41$	$4.24 \pm 0.49$	$4.18 \pm 0.49$	$4.8 \pm 2.0$
phase of $g_{\phi\omega\pi^0}/g_{\omega\rho\pi^0}$ [degr]	$-50.90 \pm 3.63$	$-61.91 \pm 3.05$	$-52.38 \pm 3.27$	$-47.92 \pm 3.52$	$-49 \pm 7.07$
coupling $g_{\phi\rho\pi^0}$ GeV $^{-1}$	$0.802 \pm 0.026$	$0.799 \pm 0.026$	$0.803 \pm 0.026$	$0.803 \pm 0.026$	$0.815 \pm 0.021$
phase of $g_{\phi\rho\pi^0}/g_{\omega\rho\pi^0}$ [degr]	$19.28 \pm 4.71$	$0.22 \pm 0.11$	$27.98 \pm 3.53$	$23.98 \pm 3.47$	—
coupling $g_{\omega\rho\pi^0}$ GeV $^{-1}$	$13.14 \pm 0.09$	$13.14 \pm 0.09$	$13.09 \pm 0.081$	$13.14 \pm 0.09$	(see text) $11.7 \div 16.1$
$\chi^2/\text{dof}$	12.88/16	17.06/16	20.94/17	12.59/16	
Probability	63%	38%	23%	70%	

It could also be accessed from  $e^+e^- \rightarrow \omega\pi^0$  data but nothing is reported in this respect [41]. Such information, if reliable, could have been valuable as it could dismiss at least one of the fit strategies (see Table 3).

Finally, the coupling constant  $g_{\rho\omega\pi}$  is found consistent with real and its value falls indeed in the expected range [41]. It is found slightly but significantly smaller than the value preferred by [40] (14.3 GeV $^{-1}$ ). Its value is however extremely stable in all fits we attempted and looks accurate; it should be noted that this parameter is only marginally influenced by isospin symmetry breaking and follows essentially from the set of radiative and leptonic decays.

The decay rates for  $\omega/\phi \rightarrow \pi^+\pi^-\pi^0$  are, of course, determined by  $g_{(\phi/\omega)\rho\pi}$  coupling constants and a model for the  $\rho$  propagator and the  $\rho \rightarrow \pi\pi$  decay amplitude. Therefore, our model can be considered as giving a good description of these, up to effects related with modelling the  $\rho$  meson propagator for phenomenological purposes.

### 7.3.4 $\phi \rightarrow K\bar{K}$ decay modes

In all attempts we have performed, a non-negligible contribution to the  $\chi^2$  comes from both  $\phi \rightarrow K\bar{K}$  decay modes. Whatever the strategy, the charged mode contributes to the  $\chi^2$  by 2.2 and the neutral mode by 1.9. However, when taking into account all sources of errors,

Table 1, clearly shows that the disagreement with reported data is not really dramatic.

On the other hand, it is admitted that model predictions for  $\phi \rightarrow K^+K^-$  have to be corrected for Coulomb interactions [59, 38], which was not done above. It has been recently shown [60] that there is a slight discrepancy between the branching fractions for charged and neutral decay modes (about  $2\sigma$ ) and that, accounting for Coulomb interactions among the (very) slow charged kaons, increases this discrepancy to  $3\sigma$ .

Aware of this question, we have redone our fits by removing  $\phi \rightarrow K^+K^-$  from the fit data set; in this case we reached a fit quality of  $\chi^2/\text{dof} = 9.04/15$  (88% probability). For symmetry, we have tried removing instead  $\phi \rightarrow K^0\bar{K}^0$ ; we reached a fit quality of  $\chi^2/\text{dof} = 9.94/15$  (82% probability). Trying to correct the model coupling constant as indicated in [59] only degraded the nominal fit quality. Therefore, we confirm in an independent way the problem raised by A. Bramon et al. [60].

In order to identify a (possible) faulty measurement, we have redone our fits by removing both  $\phi$  modes from our fits. Focusing still on the model as given in the fourth data column in Table 1, we reach a fit quality of  $\chi^2/\text{dof} = 8.42/14$  (probability 87%). What is interesting here is to consider the  $\chi^2$  distance of the measurements to what is predicted by our model by relying only on the rest of the data (24 measurements). We got  $\chi^2 = 3.76$  for the charged mode (a  $2\sigma$  effect as pointed out by [60]), while the neutral

mode yields  $\chi^2 = 0.90$ . When correcting the prediction for the charged mode by the Coulomb factor its  $\chi^2$  distance increased to  $\chi^2 = 16.6$ , a  $4\sigma$  deviation.

Therefore, we confirm the issue raised by Bramon et al. [60], with an additional information: if one among  $\phi \rightarrow K^+K^-$  and  $\phi \rightarrow K^0\bar{K}^0$  is faulty, it should be the former, which seems overestimated<sup>16</sup>. Indeed, if we correct the model coupling constants in order to account for Coulomb correction (1.042 for the rate), the global fit quality sharply degrades ( $\chi^2/\text{dof} = 21.40/16$ , probability 16%).

#### 7.4 The value of $f_K/f_\pi$

In all fits referred to above we have fixed the ratio  $f_K/f_\pi$  at the central value recommended par the Particle Data Group [34] ( $f_K/f_\pi = 1.226 \pm 0.012$ ) and neglected its error. This corresponds to using  $[f_\pi/f_K]^2 = Z = 2/3$ .

Instead of leaving it fixed, we allowed this ratio to vary in all conditions described in Sect. 7.1 and reported in Table 1. The best fits thus obtained have improved  $\chi^2$  with respect to Table 1 by only  $\simeq 0.5$  and have one less degree of freedom. The different values obtained for  $Z$  never differ by more than a per mil and can be summarized by:

$$\frac{f_K}{f_\pi} = 1.229 \pm 0.008 \quad (37)$$

which compares quite well with the PDG reference value for this datum recalled above. This result is at  $2.8\sigma$  of the recent value [6] extracted from  $Ke_3$  decay, neutron decay and nuclear Fermi transition data ( $f_K/f_\pi = 1.189 \pm 0.012$ ). We have introduced, as fixed, the corresponding value for  $Z = 0.71$  in our fits. We never reached a probability greater than 0.5%. Looking at the various contributions to the global  $\chi^2$ , we found that it is the whole  $\phi$  sector which is the most affected. Considering the discussion in [6] about the inputs which lead to this new value for  $f_K/f_\pi$ , one might think that the origin of this inconsistency is in the nuclear or in the free neutron beta decay datum used. We conclude herefrom that the traditional PDG value for  $f_K/f_\pi$  ratio is sharply favored by the whole set of radiative decays and an improved fit value is given by (37).

## 8 Conclusion

In previous work done with other coauthors, we focused on introducing SU(3) symmetry breaking and nonet symmetry breaking within the framework of the Hidden Local Symmetry Model [20]. We introduced also the  $\omega - \phi$  mixing, generated by kaon loops effects, which does not correspond to any symmetry breakdown [20, 29]. This frame-

work, supplemented with these symmetry breaking mechanisms has been shown to provide quite a successful picture of all radiative and leptonic decays of light vector and pseudoscalar mesons accessible from inside the VMD framework. We have also shown that this framework was able to explain the main features of the  $\eta - \eta'$  mixing phenomenon [21] in perfect agreement with all expectations of Chiral Perturbation Theory (ChPT); this led us to get a relation between the pseudoscalar (wave function) mixing angle, basically at work in VMD modelling ( $\simeq -10^\circ$ ), and the ChPT mixing angle recently renamed  $\theta_8$  ( $\simeq -20^\circ$ ).

In the present work, we have shown that isospin symmetry breaking can be accounted for within an effective HLS model by means of – essentially – kaon loop effects. In contrast with the case of  $\omega_I - \phi_I$  mixing where both kaon loops (charged and neutral) come additively, in the case of  $\rho_I - \omega_I$  and  $\rho_I - \phi_I$  mixings, it is their difference which occurs. Relying on the properties of Dispersion Relations, this difference should be essentially a polynomial in  $s$  with real coefficients, which is additionally constrained to vanish at  $s = 0$ . We argued that this polynomial should not be identically zero, at least to account for isospin symmetry breaking in the pseudoscalar sector. Indeed, when isospin symmetry is not broken, it is quite legitimate to choose the same renormalization conditions for both the  $K^+K^-$  and  $K^0\bar{K}^0$  loops; instead, when isospin symmetry is broken, this requirement has certainly to be relaxed.

Other mechanisms than kaon loops could also be imagined. If they play by generating  $\rho_I - \omega_I$  and  $\rho_I - \phi_I$  transition amplitudes, the angle formalism we presented here still applies without any change. However, we have shown on the pion form factor, that all properties expected from isospin symmetry breaking are strikingly reproduced by the kaon loop mechanism we advocate. We then naturally recover all properties traditionally expected from the  $\rho - \omega$  mixing amplitude:  $\Pi_{\rho_I\omega_I}(s)$  is practically real in the  $\rho - \omega$  peak invariant mass region, it is  $s$ -dependent and vanishes at the chiral point.

Moreover, we were able to derive the pion form factor in the Orsay phase formulation from our (effective) broken Lagrangian; the Orsay phase was shown to be strictly equivalent to a “rotation” by a complex angle, additionally close to purely imaginary.

Using this framework, it has been possible to extend our breaking scheme in order to include isospin symmetry breaking. Actually, taking into account the various orders of magnitude of the breaking parameters and of the  $\omega - \phi$  mixing, it is mathematically safer to define a full mixing scheme involving the triplet  $\rho$ ,  $\omega$ ,  $\phi$  as a whole. This leads us to define a priori a  $s$ -dependent rotation matrix, depending on three angles which can be real or complex.

We have thus formulated an effective Lagrangian model which is able to account quite successfully for practically all physics quantities related to VMD: radiative decays ( $VP\gamma$ ,  $P\gamma\gamma$ ), leptonic decays ( $Ve^+e^-$ ),  $VVP$  couplings, and all decays related with isospin symmetry breaking ( $\omega/\phi \rightarrow \pi\pi$ ,  $\phi \rightarrow \omega\pi$ ) in modulus and in phase. This represents 26 physics quantities all well reconstructed.

<sup>16</sup> It is interesting to note that systematics on  $\phi \rightarrow K^+K^-$  are harder to estimate than those on  $\phi \rightarrow K_L K_S$ , because the modelling of nuclear interactions of low energy charged kaons is not still fully satisfactory. Instead, the signature of  $K_S \rightarrow \pi^+\pi^-$  is much cleaner

It should be noted that all results we previously obtained without introducing isospin symmetry breaking are confirmed, including the  $\eta - \eta'$  and  $\omega - \phi$  mixing angles.

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## Appendices

In order to be self-contained, we collect in this Appendix formulae for coupling constants and partial widths; we do not insist much on how U(3)/SU(3) breaking is performed in the present paper, as it is the matter of already published work [18,29,20,21] to which the interested reader can refer.

### A Details of the Breaking Model

Our framework is the HLS model and the SU(3) breaking procedure we follow has been defined first in [19,18]. Focusing on the anomalous sector [15], all details can be found in [20,29,21]. Here, we mainly recall breaking parameter properties or values of concern for the present study.

Breaking the non-anomalous sector of the HLS model [14,19,18] introduces a breaking parameter  $Z$  strongly associated with the pseudoscalar (PS) sector; it is not a free parameter but fulfills  $Z = [f_\pi/f_K]^2 = 2/3$ .

Concerning the PS sector, we have a priori 2 additional parameters. The first is named  $x$  and its departure from 1 measures breaking of nonet symmetry in the PS sector. Another parameter affecting the PS sector is the PS mixing angle  $\theta_P$  (which describes the  $\eta/\eta'$  sector in terms of mixtures of singlet and octet components) or  $\delta_P$  (when one prefers referring to departures from ideally mixed states). Both angles are used and are trivially related to each other [20].

When studying the connection between VMD, the Wess-Zumino-Witten Lagrangian and Chiral Perturbation Theory, it has been found [21] that the PS mixing angle  $\theta_P$  and the nonet symmetry breaking parameter  $x$  fulfill:

$$\tan \theta_P = \sqrt{2} \frac{Z-1}{2Z+1} x \quad (\text{A1})$$

with high accuracy (in [21], this relation is given in terms of  $z = 1/Z$ ). Preliminary fits have shown that this relation is still satisfied in the present framework without

any degradation; thus it is assumed. We remind that the mixing angle  $\theta_P$  relates to the (now) more usual ChPT mixing angle  $\theta_8$  by [21]  $\theta_8 \simeq 2\theta_P$ . Therefore, concerning the PS sector, our model depends only on one free parameter which can be either of  $\theta_P$  or  $x$ . We choose the former.

Associated with the vector sector and, more precisely with vector meson masses, another breaking parameter occurs named here  $\ell_V$ ; it relates with another breaking parameter ( $c_V$ ) [19,18] by  $\ell_V = (1 + c_V)^2$ . It is a priori subject to fit, and thus free, as the connection between reported vector meson masses [34] and the corresponding masses occurring in the HLS Lagrangian is unclear [29].

Concerning vector mesons, another breaking parameter is necessary in order to account for the anomalous  $K^*$  sector; it is named [20,29]  $\ell_T$ . It was first considered as somewhat ad hoc [20]; however, it has been shown [29] that it strictly corresponds within VMD to a breaking parameter defined independently by G. Morpurgo [49] within the non-relativistic quark model and found in agreement with low energy QCD. If the partial width value for  $K^{*\pm} \rightarrow K^\pm \gamma$  is confirmed, this parameter looks unavoidable within VMD; its precise meaning is still to be understood [29].

A possible break up of nonet symmetry in the vector sector has been found previously undetectable (the parameter  $y$  defined and studied in [29]). Preliminary fits in the present study confirmed this conclusion and, therefore, the parameter  $y$  was set to 1 definitely.

Thus, concerning SU(3) symmetry breaking, the HLS vector sector depends already on 2 free parameters  $\ell_V$  and  $\ell_T$ , independently of mixing among the ideal field combinations  $\omega_I$ ,  $\phi_I$  and  $\rho_I$  associated with neutral vector mesons. This last point is the actual subject of the present paper.

### B Basic Coupling Constants and Partial widths

We give in this Section all coupling constants which cannot be trivially read off the Lagrangian pieces given in the main text.

#### B.1 Radiative Decays

Starting from the Lagrangian in (28), and using the breaking procedure as defined in [29,20,21], one can compute the coupling constants for all radiative and leptonic decays of relevance. Let us define:

$$G = -\frac{3eg}{8\pi^2 f_\pi}, \quad G' = -\frac{3eg}{8\pi^2 f_K}, \quad Z = [f_\pi/f_K]^2. \quad (\text{B1})$$

Some  $VP\gamma$  coupling constants are not affected by the parameters of isospin symmetry breaking. These are:

$$\begin{cases} G_{\rho^\pm \pi^\pm \gamma} = \frac{1}{3} G \\ G_{K^{*0} K^0 \gamma} = -\frac{3}{3} \sqrt{\ell_T} \left(1 + \frac{1}{\ell_T}\right) \\ G_{K^{*\pm} K^\pm \gamma} = \frac{G'}{3} \sqrt{\ell_T} \left(2 - \frac{1}{\ell_T}\right). \end{cases} \quad (\text{B2})$$

The  $\rho_I P \gamma$  coupling constants are:

$$\begin{cases} G_{\rho_I \pi^0 \gamma} = \frac{1}{3}G \\ G_{\rho_I \eta \gamma} = \frac{1}{3}G \left[ \sqrt{2}(1-x) \cos \delta_P - (2x+1) \sin \delta_P \right] \\ G_{\rho_I \eta' \gamma} = \frac{1}{3}G \left[ \sqrt{2}(1-x) \sin \delta_P + (2x+1) \cos \delta_P \right] \end{cases} \quad (\text{B3})$$

The other single photon radiative modes provide the following coupling constants:

$$\begin{cases} G_{\omega_I \pi^0 \gamma} = G \\ G_{\phi_I \pi^0 \gamma} = 0 \\ G_{\omega_I \eta \gamma} = \frac{1}{9}G \left[ \sqrt{2}(1-x) \cos \delta_P - (1+2x) \sin \delta_P \right] \\ G_{\omega_I \eta' \gamma} = \frac{1}{9}G \left[ (1+2x) \cos \delta_P + \sqrt{2}(1-x) \sin \delta_P \right] \\ G_{\phi_I \eta \gamma} = -\frac{2}{9}G \left[ Z(2+x) \cos \delta_P - \sqrt{2}Z(1-x) \sin \delta_P \right] \\ G_{\phi_I \eta' \gamma} = -\frac{2}{9}G \left[ \sqrt{2}Z(1-x) \cos \delta_P + Z(2+x) \sin \delta_P \right] \end{cases} \quad (\text{B4})$$

## B.2 $P \gamma \gamma$ and $V - \gamma$ Modes

The 2-photon decay modes are not affected by isospin symmetry breaking in the vector sector and keep their usual form within the HLS model [20,29,21]:

$$\begin{cases} G_{\eta \gamma \gamma} = -\frac{\alpha_{em}}{\pi \sqrt{3} f_\pi} \left[ \frac{5-2Z}{3} \cos \theta_P - \sqrt{2} \frac{5+Z}{3} x \sin \theta_P \right] \\ G_{\eta' \gamma \gamma} = -\frac{\alpha_{em}}{\pi \sqrt{3} f_\pi} \left[ \frac{5-2Z}{3} \sin \theta_P + \sqrt{2} \frac{5+Z}{3} x \cos \theta_P \right] \\ G_{\pi^0 \gamma \gamma} = -\frac{\alpha_{em}}{\pi f_\pi} \end{cases} \quad (\text{B5})$$

As stated in the text, we actually replace this last coupling by the world average value for  $f_\pi$  as given in the RPP [34].

Finally, the leptonic decay widths of vector mesons depend on the HLS  $V - \gamma$  couplings. For the ideal combinations, we have:

$$\begin{cases} f_{\rho_I \gamma} = a f_\pi^2 g \\ f_{\omega_I \gamma} = \frac{f_{\rho_I \gamma}}{3} \\ f_{\phi_I \gamma} = \frac{f_{\rho_I \gamma}}{3} \sqrt{2} \ell_V \end{cases} \quad (\text{B6})$$

## B.3 Partial widths

We list for completeness in this Section the expressions for the partial widths in terms of the coupling constants for the various cases.

The two-photon partial widths are:

$$\Gamma(P \rightarrow \gamma \gamma) = \frac{m_P^3}{64\pi} |G_{P\gamma\gamma}|^2, \quad P = \pi^0, \eta, \eta'. \quad (\text{B7})$$

The leptonic partial widths are:

$$\Gamma(V \rightarrow e^+ e^-) = \frac{4\pi\alpha^2}{3m_V^3} |f_{V\gamma}|^2. \quad (\text{B8})$$

The radiative widths are:

$$\Gamma(V \rightarrow P\gamma) = \frac{1}{96\pi} \left[ \frac{m_V^2 - m_P^2}{m_V} \right]^3 |G_{VP\gamma}|^2, \quad (\text{B9})$$

where  $V$  is either of  $\rho^0, \omega, \phi$  and  $P$  is either of  $\pi^0, \eta, \eta'$ , and:

$$\Gamma(P \rightarrow V\gamma) = \frac{1}{32\pi} \left[ \frac{m_P^2 - m_V^2}{m_P} \right]^3 |G_{VP\gamma}|^2. \quad (\text{B10})$$

The decay width for a vector meson decaying to  $V + P$  is:

$$\begin{aligned} \Gamma(V' \rightarrow VP) &= \frac{1}{96\pi} \left[ \frac{\sqrt{[m_{V'}^2 - (m_V + m_P)^2][m_{V'}^2 - (m_V - m_P)^2]}}{m_{V'}} \right]^3 \\ &\times |G_{V'VP}|^2. \end{aligned} \quad (\text{B11})$$

Finally, the partial width for a vector meson decaying into two pseudoscalar mesons of equal masses is:

$$\Gamma(V \rightarrow PP) = \frac{1}{48\pi} \frac{[m_V^2 - 4m_P^2]^{3/2}}{m_V^2} |G_{VPP}|^2. \quad (\text{B12})$$

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